Exponents

Integer Exponents and Scientific Notation

- **Exponents**: An exponent is repeated multiplication of the same number. If *x* is a real number and *n* is a natural number, then $x^n = x \cdot x \cdot x \cdots x$ where x is multiplied *n* times. *n* is called the exponent or power and *x* is called the base.

  $$5^4 = 5 \cdot 5 \cdot 5 \cdot 5 = 625 \qquad \text{or} \qquad x^5 = x \cdot x \cdot x \cdot x \cdot x$$

  NOTE: For order of operations, we now include exponents so the order is as follows: (1) Parentheses (2) Exponents (3) Multiplication/division (4) Addition/subtraction
- **Properties of Exponents:**
  - Negative- Integer Exponent Property: If *x* is any nonzero real number and *n* is a natural number, then $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$.
  - Zero- Exponent Property: If x is any nonzero real number, then $x^0 = 1$.
  - Product Property: When multiplying exponentials with the same base, add the exponents.

    $$x^3 \cdot x^4 = x^{3+4} = x^7$$
  - Quotient Property: When dividing exponentials with the same base, subtract the exponents.

    $$\frac{x^6}{x^4} = x^{6-4} = x^2 \qquad \text{or} \qquad \frac{x^3}{x^7} = \frac{1}{x^{7-3}} = \frac{1}{x^4} = x^{-4}$$
  - Power Property: When raising an exponential to a power, multiply exponents.

    $$(x^5)^2 = x^{5\cdot 2} = x^{10}$$
  - Product to a Power Property: A product raised to a power is equal to the product of each factor raised to the power.

    $$(3x)^2 = 3^2 \cdot x^2 = 9x^2$$
  - Quotient to a Power Property: A quotient raised to a power is equal to the quotient of the factors raised to the power.

    $$\left(\frac{x}{3}\right)^3 = \frac{x^3}{3^3} = \frac{x^3}{27}$$

NOTE: An exponential expression is simplified when all (1) all parentheses have been eliminated (2) a base appears only once (3) No powers are raised to other powers (4) all exponents are positive.

Example: Simplify the following expression: $(2x^2y^3)^2 \cdot \frac{3x}{y^{10}} =$

$$(2^2 \cdot x^{2\cdot 2} \cdot y^{3\cdot 2}) \cdot \frac{3x}{y^{10}} = 4x^4y^6 \cdot \frac{3x}{y^{10}} = \frac{(4x^4y^6)(3x)}{y^{10}} = \frac{(4\cdot 3)(x^{4+1})(y^6)}{y^{10}} = \frac{12x^5y^6}{y^{10}} = \frac{12x^5}{y^{10-6}} = \frac{12x^5}{y^4}$$

- **Scientific Notation:** Very small or very large real numbers often are written using scientific notation which has the form $c \times 10^n$, where $1 \le c < 10$ and n is an integer.

  $456{,}920{,}000{,}000 = 4.5692 \times 10^{11}$

  $0.00000521 = 5.21 \times 10^{6}$

Rational Exponents and Radicals

Key Definitions

- **<u>Square Root</u>:** A square root of a nonnegative real number a is the nonnegative number b where $= \sqrt{a}$ if $b^2 = a$.

  Example: Evaluate the square root: $\sqrt{49} = 7$

Nth Root

- **<u>Topic</u>:** An nth root of a real number a is the real number b such that $b = \sqrt[n]{a}$ if $b^n = a$ where n is a positive integer. If n is even, then both a and b are nonnegative real numbers.

  Examples: Simplify: $\sqrt[5]{-32} = -2$ Simplify: $\sqrt[4]{81} = 9$

  Note: A square root is an nth root where $n = 2$.

Properties of Radicals

- **<u>Product Property</u>:** The nth root of a product is the product of the nth roots.

$$\sqrt[4]{32} = \sqrt[4]{16 \cdot 2} = \sqrt[4]{16} \cdot \sqrt[4]{2} = 2\sqrt[4]{2}$$

- **<u>Quotient Property</u>:** The nth root of a quotient is the quotient of the nth roots.

$$\sqrt[3]{\frac{27}{8}} = \frac{\sqrt[3]{27}}{\sqrt[3]{8}} = \frac{3}{2}$$

- **<u>Power Property</u>:** The nth root of a power is the power of the nth root.

$$\sqrt[4]{16^2} = (\sqrt[4]{16})^2 = (2)^2 = 4$$

- **<u>Index Properties</u>:**
  - When n is odd, the nth root of a raised to the nth power is a.

$$\sqrt[3]{x^7} = \sqrt[3]{x^3} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x} = x \cdot x \cdot \sqrt[3]{x} = x^2\sqrt[3]{x}$$

  - When n is even, the nth root of a raised to the nth power is the absolute value of a.

$$\sqrt[4]{x^6} = \sqrt[4]{x^4} \cdot \sqrt[4]{x^2} = |x| \cdot \sqrt[4]{x^2} = x \cdot \sqrt[4]{x^2} = x\sqrt[4]{x^2}$$

Rational Exponents

- Rational exponents were defined in terms of radicals: $a^{\frac{1}{n}} = \sqrt[n]{a}$. The properties for integer exponents from Section 0.2 hold true for rational exponents:

$$a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m \text{ or } a^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m}$$

  Note: Negative rational exponents: $a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{(\sqrt[n]{a})^m}$ $or = \frac{1}{\sqrt[n]{a^m}}$ $where\ a \neq 0$